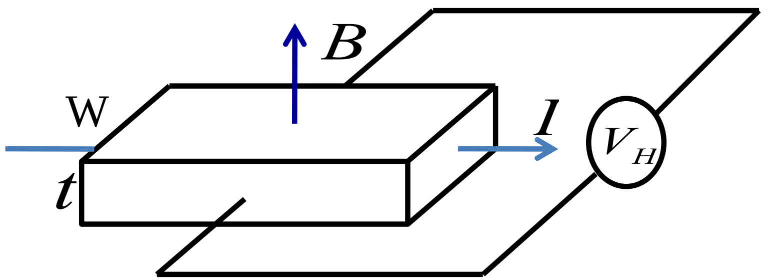
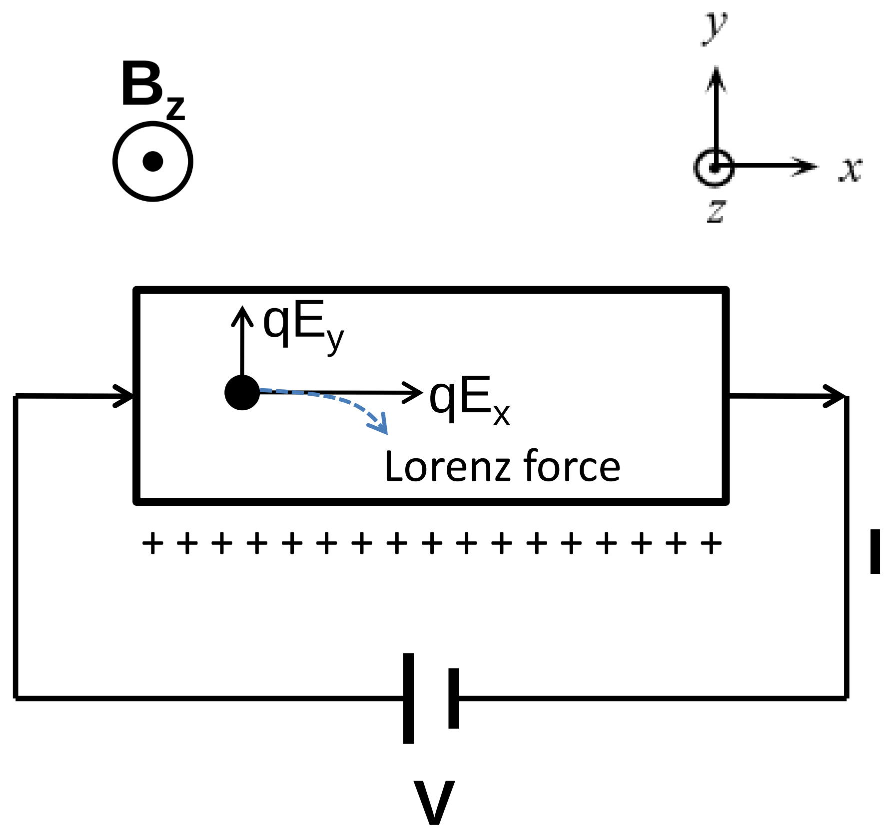
Hall effect in semiconductors



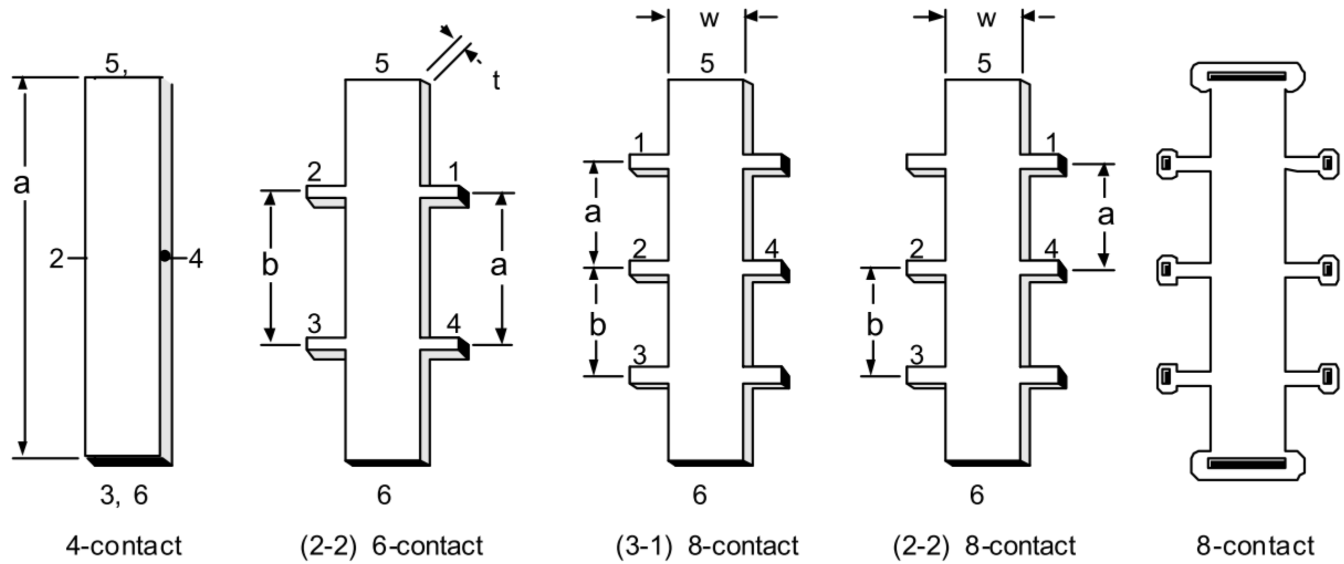


$$\begin{cases} qv_xB = qE_y \Rightarrow v_xB = E_y = \dfrac{V_H}{W} \Rightarrow V_H = v_xBW \\ J_x = \dfrac{I_x}{Wt} = nv_x \Rightarrow v_x = \dfrac{I_x}{nWt} \end{cases}$$

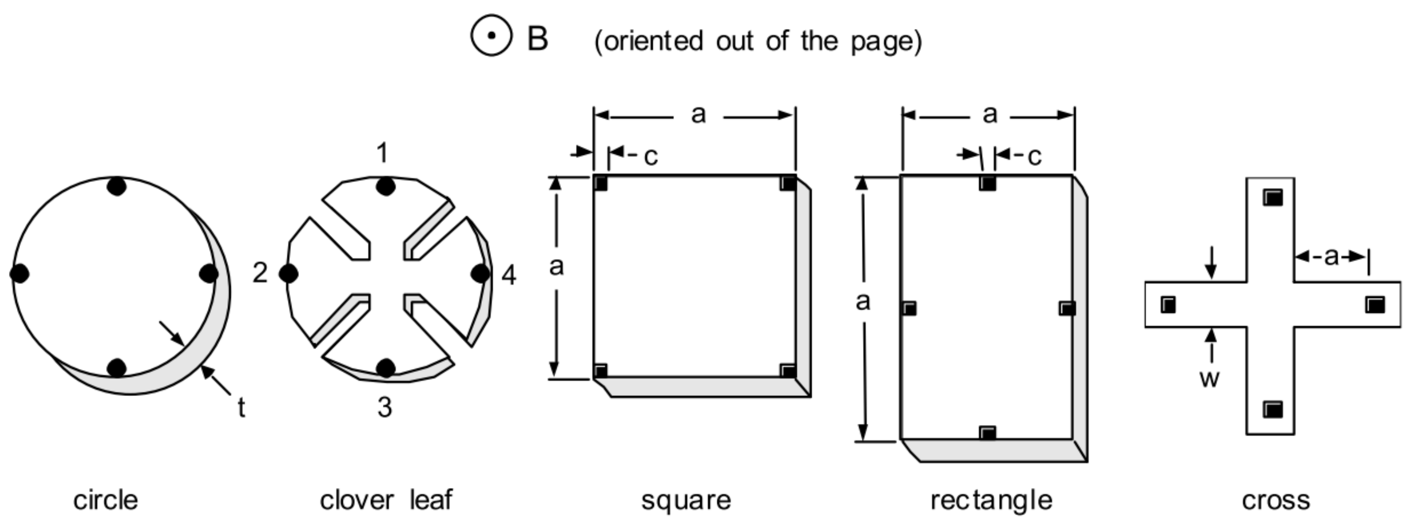
$$\Rightarrow R_H \equiv \frac{tV_H}{BI_x} = \frac{1}{qn_H}$$

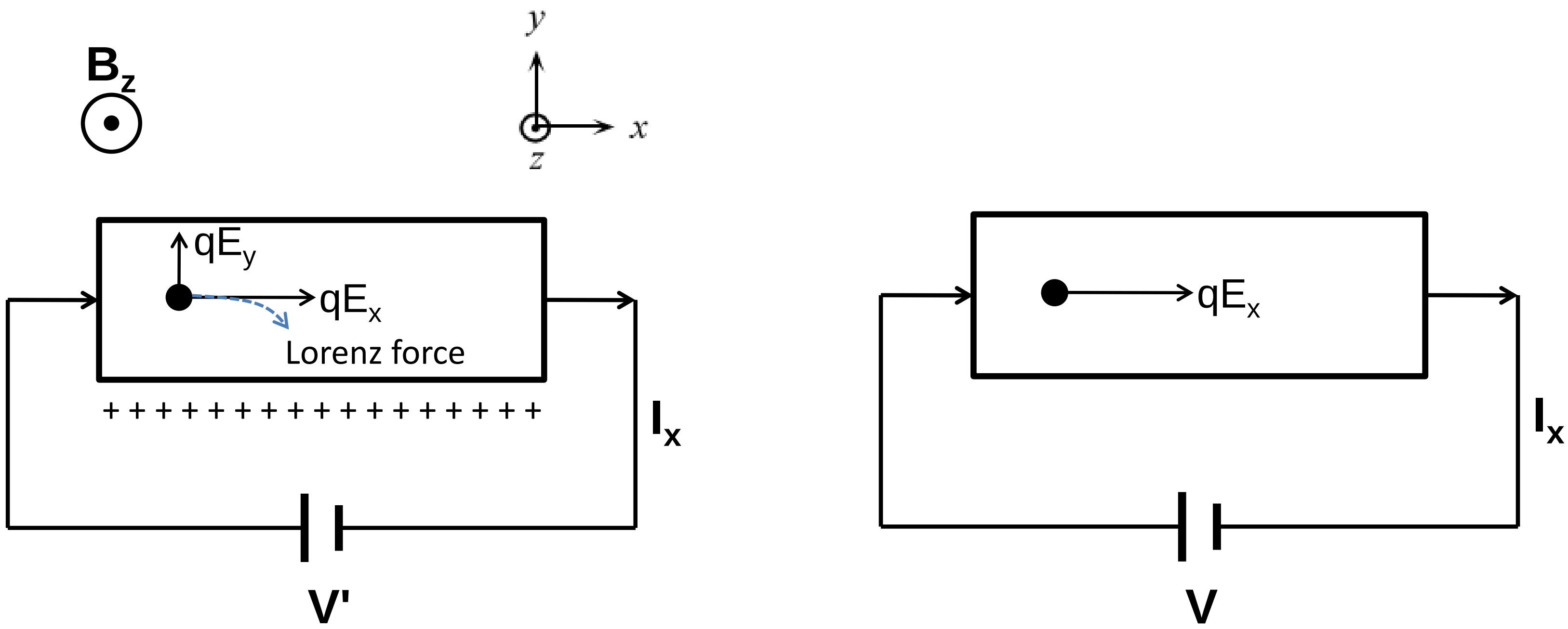
$$\Rightarrow n_H = \frac{1}{qR_H} = \frac{BI_x}{qtV_H}$$

1. <u>Hall bar geometry</u>：



2. <u>Van der Pauw geometry</u>：





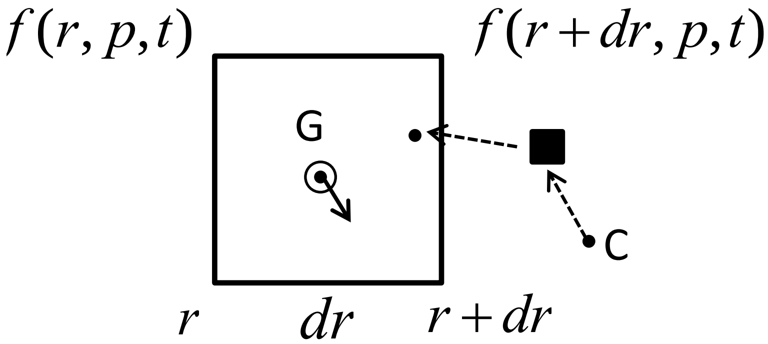
Scattering mechanisms	Energy dependence	Hall factor
Lattice scattering	-1/2	1.18
Ionized impurity(weakly screened)	3/2	1.93
Ionized impurity(strongly screened)	-1/2	1.18
Neutral impurity	0	1
Piezoelectric	1/2	1.10

Hall coefficient **Hall factor**

$$R_H \equiv \frac{tV_H}{BI} = \frac{r_H(\sim 1)}{en}$$

Carrier conc.

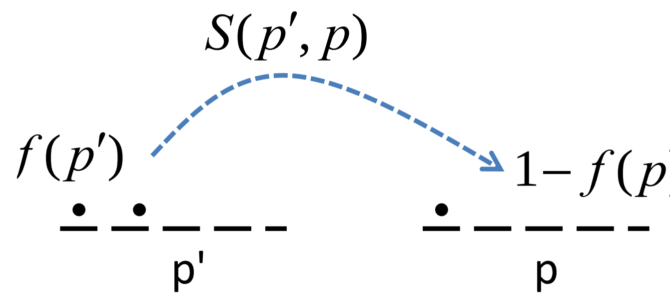
$$r_H \equiv \langle \tau_m{}^2 \rangle / \langle \tau_m \rangle^2 \qquad \langle \tau_m{}^n \rangle = \frac{\int_0^\infty \tau_m{}^n \varepsilon g(\varepsilon) \frac{\partial f_0}{\partial \varepsilon} d\varepsilon}{\int_0^\infty \varepsilon g(\varepsilon) \frac{\partial f_0}{\partial \varepsilon} d\varepsilon}$$



$$df = f(r+dr,p,t) - f(r,p,t)$$
$$= \frac{\partial f}{\partial r} dr$$

$$\Rightarrow \frac{df}{dt} = \frac{\partial f}{\partial r}\frac{dr}{dt} = \frac{\partial f}{\partial r} v$$

$$\Rightarrow \frac{df}{dt} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f + F \cdot \nabla_p f + G + C$$



Steady state $\frac{\partial f}{\partial t} + v \cdot \nabla_r f + F \cdot \nabla_p f = \left.\frac{\partial f}{\partial t}\right|_{coll}$

$$\left.\frac{\partial f(p)}{\partial t}\right|_{coll} = \sum_{p'} S(p',p) f(p')(1-f(p)) - \sum_{p'} S(p,p') f(p)(1-f(p'))$$

$$\text{Steady state}\ \frac{\partial f}{\partial t} + v \cdot \nabla_r f + F \cdot \nabla_p f = \left.\frac{\partial f}{\partial t}\right|_{coll}$$

$$\left.\frac{\partial f(p)}{\partial t}\right|_{coll} = \sum_{p'} S(p', p) f(p')(1 - f(p)) - \sum_{p'} S(p, p') f(p)(1 - f(p')) = -\frac{f - f_0}{\tau_m}$$

等向性
彈性

$$(v \cdot \nabla_r f) + e\hbar^{-1}\{(E \cdot \nabla_k f) + ([v \times B] \cdot \nabla_k f\} + (f - f_0)/\tau_m = 0$$

$$\Rightarrow f = f_0 - \frac{df_0}{d\varepsilon} e\hbar^{-1} \nabla_k \varepsilon \tau_m \frac{F - e[B \times m^{-1} \tau_m F] + \mu^2 B(F \cdot B)}{1 + \mu^2 B^2}$$

對稱項 非對稱項

$$\text{drift velocity}\ v_d = \frac{q < \tau_m >}{m^*} = \int_{-\infty}^{\infty} v f d^3 v \Bigg/ \int_{-\infty}^{\infty} f d^3 v$$

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \begin{pmatrix} \sigma - \beta B_x^2 & \gamma B_z - \beta B_x B_y & -\gamma B_y - \beta B_x B_z \\ -\gamma B_z - \beta B_x B_y & \sigma - \beta B_y^2 & \gamma B_x - \beta B_y B_z \\ \gamma B_y - \beta B_x B_z & -\gamma B_x - \beta B_y B_z & \sigma - \beta B_z^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\sigma = (ne^2/m)\left\langle \tau_m/(1+\omega_c^2\tau_m^2)\right\rangle \qquad \gamma = (ne^3/m^2)\left\langle \tau_m^2/(1+\omega_c^2\tau_m^2)\right\rangle \qquad \omega_c = |e|B/m$$

$$\beta = -(ne^4/m^3)\left\langle \tau_m^3/(1+\omega_c^2\tau_m^2)\right\rangle$$

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \begin{pmatrix} \sigma_0 + \beta_0(B_y^2 + B_z^2) & \gamma_0 B_z - \beta_0 B_x B_y & -\gamma_0 B_y - \beta_0 B_x B_z \\ -\gamma_0 B_z - \beta_0 B_x B_y & \sigma_0 + \beta_0(B_x^2 + B_z^2) & \gamma_0 B_x - \beta_0 B_y B_z \\ \gamma_0 B_y - \beta_0 B_x B_z & -\gamma_0 B_x - \beta_0 B_y B_z & \sigma_0 + \beta_0(B_x^2 + B_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\sigma_0 = (ne^2/m)\langle \tau_m \rangle \qquad \gamma_0 = (ne^3/m^2)\langle \tau_m^2 \rangle \qquad \beta_0 = -(ne^4/m^3)\langle \tau_m^3 \rangle$$

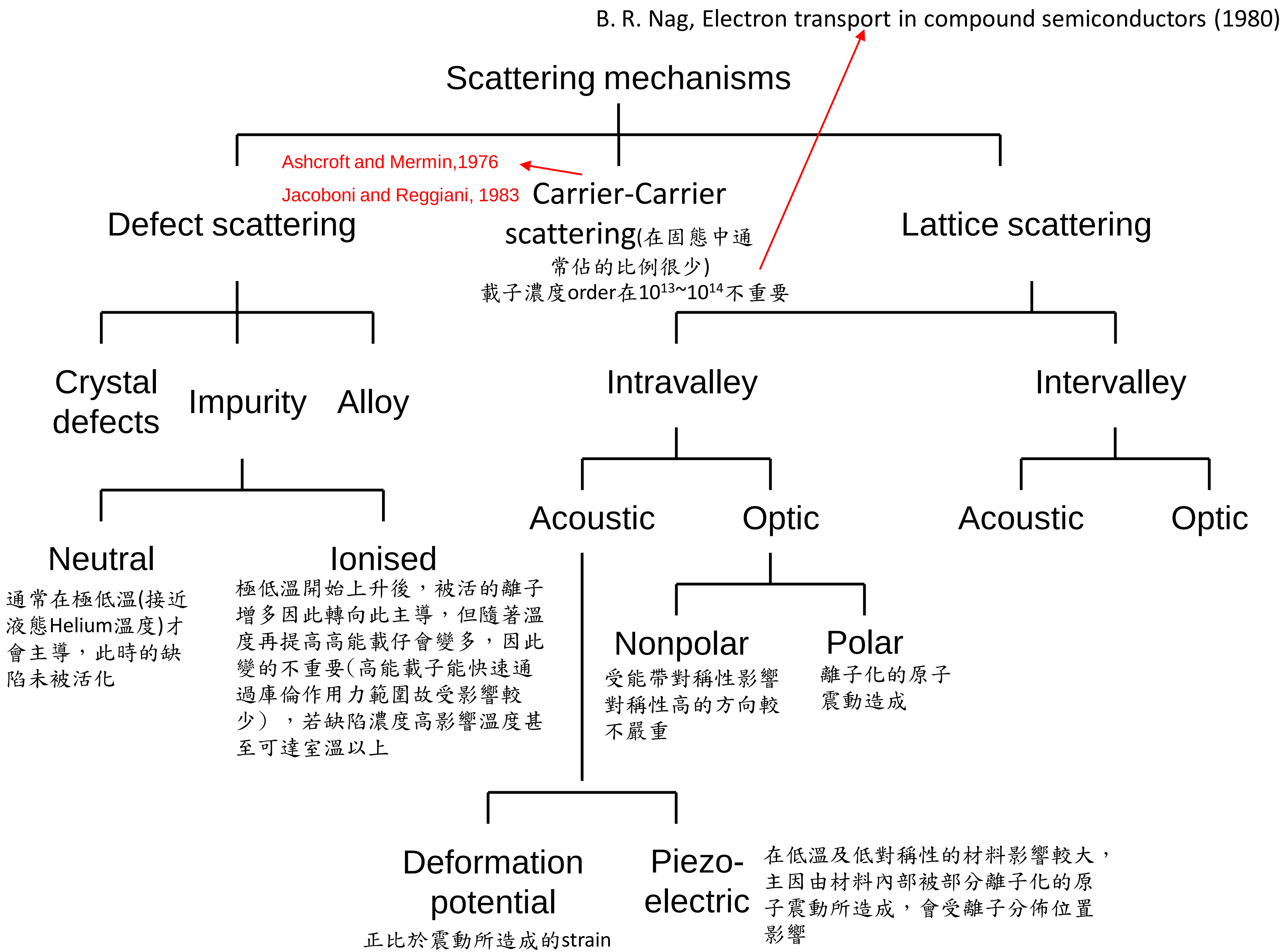
$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \begin{pmatrix} \sigma_0 + \beta_0(B_y^2 + B_z^2) & \gamma_0 B_z - \beta_0 B_x B_y & -\gamma_0 B_y - \beta_0 B_x B_z \\ -\gamma_0 B_z - \beta_0 B_x B_y & \sigma_0 + \beta_0(B_x^2 + B_z^2) & \gamma_0 B_x - \beta_0 B_y B_z \\ \gamma_0 B_y - \beta_0 B_x B_z & -\gamma_0 B_x - \beta_0 B_y B_z & \sigma_0 + \beta_0(B_x^2 + B_y^2) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$\sigma_0 = (ne^2/m)\langle \tau_m \rangle \qquad \gamma_0 = (ne^3/m^2)\langle \tau_m^2 \rangle \qquad \beta_0 = -(ne^4/m^3)\langle \tau_m^3 \rangle$$

$$\begin{cases} j_x = (\sigma_0 + \beta_0 B_z^2) E_x + \gamma_0 B_z E_y \\ j_y = -\gamma_0 B_z E_x + (\sigma_0 + \beta_0 B_z^2) E_y = 0 \end{cases}$$

$$R_H \equiv \frac{tV_{H,y}}{BI_x} = \frac{E_y}{j_x B_z} = \frac{\gamma_0}{(\sigma_0 + \beta_0 B_z^2)^2 + \gamma_0^2 B_z^2} \approx \gamma_0 / \sigma_0^2 = \frac{r_H}{ne}$$

1. M. Lundstrom, *Fundamentals of Carrier Transport* (Cambridge University Press, New York, 2000).
2. K. Seeger, *Semiconductor Physics: An Introduction* (Springer, New York, 2004).


B. R. Nag, Electron transport in compound semiconductors (1980)
Scattering mechanisms
Ashcroft and Mermin,1976
Jacoboni and Reggiani, 1983
Carrier-Carrier scattering(在固態中通常佔的比例很少)
載子濃度order在10^{13}~10^{14}不重要
Defect scattering
Lattice scattering
Crystal defects
Impurity
Alloy
Intravalley
Intervalley
Acoustic
Optic
Acoustic
Optic
Neutral
通常在極低溫(接近液態Helium溫度)才會主導，此時的缺陷未被活化
Ionised
極低溫開始上升後，被活的離子增多因此轉向此主導，但隨著溫度再提高高能載仔會變多，因此變的不重要(高能載子能快速通過庫倫作用力範圍故受影響較少），若缺陷濃度高影響溫度甚至可達室溫以上
Nonpolar
受能帶對稱性影響對稱性高的方向較不嚴重
Polar
離子化的原子震動造成
Deformation potential
正比於震動所造成的strain
Piezo-electric
在低溫及低對稱性的材料影響較大，主因由材料內部被部分離子化的原子震動所造成，會受離子分佈位置影響